Stoichiometric Relationships Part two (answers)

IB CHEMISTRY SL/HL

25	16		6	2	25
Mn	S	J	C	He	Mn
Manganese	Sulfur		Carbon	Helium	Manganese
54.938045	32.065		12.0107	4.002602	54.938045

Syllabus objectives:

Understandings

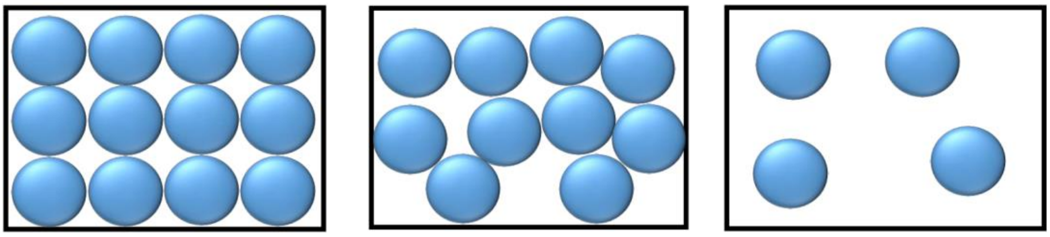
- Avogadro's law enables the mole ratio of reacting gases to be determined from volumes of the gases.
- The molar volume of an ideal gas is a constant at specified temperature and pressure.

Applications and skills:

- Calculation of reacting volumes of gases using Avogadro's law.
- Solution of problems and analysis of graphs involving the relationship between temperature, pressure and volume for a fixed mass of an ideal gas.
- Solution of problems relating to the ideal gas equation
- Explanation of the deviation of real gases from ideal behavior at low temperatures and high pressures
- Obtaining and using experimental values to calculate the molar mass of a gas from the ideal gas equation.

Introduction to gases

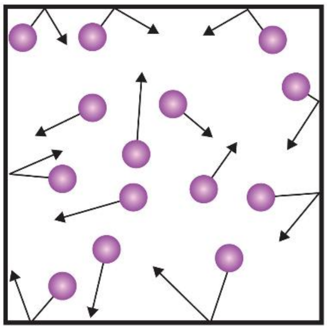
- Gases, unlike solids and liquids, do not have a definite shape or volume.
- Because of this, they are subject to changes in pressure, temperature and volume.
- The particle models of solids, liquids and gases are shown below.



Kinetic molecular theory of gases

1. A gas consists of a collection of small particles traveling in straight-line motion.
2. The molecules in a gas occupy virtually no volume.
3. Collisions between molecules are perfectly elastic (no energy is gained or lost during the collision).
4. There are no attractive or repulsive forces (intermolecular forces) between the molecules.
5. The average kinetic energy of a gas particle is directly proportional to its temperature in kelvin (K).

Gas particles in a pressurized container



- The particles are in constant motion, colliding with the walls of the container.
- The forces exerted by the particles on the walls of the container gives rise to pressure.
- Increasing the temperature increases the average kinetic energy of the particles, and pressure inside the container increases.

Ideal gases vs real gases

- An ideal gas is described as a gas that has virtually no volume, no intermolecular forces between particles and whose collisions are perfectly elastic.
- Real gases do not behave as ideal gases under two conditions; high pressure and low temperature.
- When gases are compressed to high pressure, the gas molecules come close enough for intermolecular forces to act between them.
- At high pressure the volume of a gas becomes significant.

Molar volume of a gas

- The molar volume of a gas is the volume taken up by one mole of an ideal gas under standard conditions (STP).
- At STP (273 K and 1.00×10^5 Pa), one mole of an ideal gas occupies a volume of:

$22.7\ dm^3$

- One mole of gas at STP = 6.02×10^{23} particles.

$$\text{number of moles of gas (n)} = \frac{\text{volume in dm}^3\text{(V)}}{22.7\ \text{dm}^3}$$

$$\text{volume in dm}^3\text{ (V)} = \text{number of moles of gas (n)} \times 22.7\ \text{dm}^3$$

Examples:

1) Calculate the volume occupied by 16.00 g of O_2 at STP.

1 mol gas at STP = 22.7 dm^3

Convert grams of O_2 to mol of O_2

n = m÷M

n = 16.00 ÷ 32.00 = 0.5000 mol O_2

0.5000 × 22.7 = 11.4 dm^3 O_2

2) Calculate the amount in mol (n) of 54.5 dm^3 of CH_4 at STP.

1 mol of gas at STP = 22.7 dm^3

54.5 ÷ 22.7 = 2.40 mol CH_4

Answers:

1)

a) $(0.754 \times 22.7) = 17.1\ dm^3$

b) $(0.754 \times 70.9) = 53.5g\ Cl_2$

c) $(0.754 \times 6.02 \times 10^{23}) = 4.54 \times 10^{23}$ Cl_2 molecules

d) $(4.54 \times 10^{23} \times 2) = 9.08 \times 10^{23}$ Cl atoms

2)

a) $(3.01 \times 10^{23} \div 6.02 \times 10^{23}) = 0.500$ mol O_2

b) $(0.500 \times 32.00) = 16.0g\ O_2$

c) $(0.500 \times 22.7) = 11.4\ dm^3$

d) $(3.01 \times 10^{23} \times 2) = 6.02 \times 10^{23}$ oxygen atoms

3)

a) $(25.0 \div 28.02) = 0.892$ mol N_2

b) $(0.892 \times 22.7) = 20.2\ dm^3\ N_2$

c) $(0.892 \times 6.02 \times 10^{23}) = 5.37 \times 10^{23}$ molecules N_2

4)

a) $(5.72 \times 22.7) = 1.30 \times 10^2\ dm^3$

b) $(5.72 \times 6.02 \times 10^{23}) = 3.44 \times 10^{24}$ molecules NH_3

c) $(3.44 \times 10^{24} \times 3) = 1.03 \times 10^{25}$ atoms H

5) Balanced equation: $Mg_{(s)} + 2HCl_{(aq)} \rightarrow MgCl_{2(aq)} + H_{2(g)}$

(n) Mg = $(3.54 \div 24.31) = 0.146$ mol

Ratio of Mg to H_2 is 1:1

(n) H_2 = 0.146 mol

Volume of H_2 at STP = $(0.146 \times 22.7) = 3.31\ dm^3$

6) Balanced equation: $CaCO_{3(s)} + 2HCl_{(aq)} \rightarrow CaCl_{2(aq)} + CO_{2(g)} + H_2O_{(l)}$

(n) $CaCO_3$ = $(139 \div 100.09) = 1.39$ mol

Ratio $CaCO_3$ to CO_2 is 1:1

(n) CO_2 = 1.39 mol

Volume of CO_2 at STP = $(1.39 \times 22.7) = 31.6\ dm^3$

Avogadro's law

- Equal volumes of gases at the same temperature and pressure contain the same number of particles.

At STP (273 K and 100 kPa)

amount	1 mol H_2	1 mol N_2	1 mol O_2
volume (dm^3)	22.7	22.7	22.7
mass (g)	2.02	28.02	32.00
# of particles	6.02×10^{23}	6.02×10^{23}	6.02×10^{23}

Example: 40 cm^3 of CO reacts with 40 cm^3 of O_2. What volume of CO_2 is produced? What volume of the excess reactant remains?

$$2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}$$

Molar ratio of CO to O_2 is 2:1

40 cm^3 of CO reacts with 20 cm^3 of O_2 (molar ratio is 2:1)

Volume of O_2 is 40 cm^3, therefore O_2 is excess reactant

Excess reactant remaining = 40 – 20 = 20 cm^3 of O_2

Exercises:

1. What volume of sulfur trioxide, in cm^3, can be prepared using 40 cm^3 sulfur dioxide and 20 cm^3 oxygen gas by the following reaction? Assume all volumes are measured at the same temperature and pressure.

$$2SO_{2(g)} + O_{2(g)} \rightarrow 2SO_{3(g)}$$

Ratio of SO_2 to O_2 is 2:1

40 cm^3 of SO_2 would react with 20 cm^3 of O_2 to produce 40 cm^3 of SO_3

2. 5 dm^3 of carbon monoxide, $CO_{(g)}$, and 2 dm^3 of oxygen, $O_{2(g)}$, at the same temperature and pressure are mixed together. What is the maximum volume of carbon dioxide, $CO_{2(g)}$, in dm^3, that can be formed? What volume of the excess reactant remains?

$$2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}$$

Ratio of CO to O_2 is 2:1
5 dm^3 of CO would need 5 ÷ 2 = 2.5 dm^3 of O_2 to react completely.
Therefore, O_2 is the LR and CO is the excess reactant.
Ratio of O_2 to CO_2 is 1:2, therefore 2 dm^3 of O_2 would produce 4 dm^3 of CO_2
2 dm^3 of O_2 reacts with 4 dm^3 of CO, 5-4 = 1 dm^3 of CO remains.

3. 100 cm^3 of ethene, C_2H_4, is burned in 400 cm^3 of oxygen, producing carbon dioxide and some liquid water. Some oxygen remains unreacted (excess).

$$C_2H_{4(g)} + 3O_{2(g)} \rightarrow 2CO_{2(g)} + 2H_2O_{(g)}$$

Calculate the volume of carbon dioxide produced and the volume of oxygen remaining.

Molar ratio of reactants and products is 1:3:2:2
100 cm^3 of C_2H_4 reacts with 300 cm^3 of O_2 to produce 200 cm^3 of CO_2 and 200 cm^3 of H_2O
Volume of O_2 remaining: 400 – 300 = 100 cm^3

Ideal gas equation

$$PV = nRT$$

P = pressure in Pa

V = volume in m^3

n = amount in mol

R = universal gas constant (8.31 J $K^{-1}mol^{-1}$)

T = temperature in kelvin (K)

$$n = \frac{PV}{RT} \quad V = \frac{nRT}{P} \quad P = \frac{nRT}{V}$$

$$M = \frac{mRT}{PV}$$

Unit conversions

- Temperature in kelvin (K): $^{o}C + 273$

  $25^{o}C = 298$ K
- Pressure in Pa: 1.00×10^5 Pa = 100 kPa
- 1 cm^3 = 1×10^{-3} dm^3 = 1×10^{-6} m^3
- 1 m^3 = 1×10^3 dm^3 = 1×10^6 cm^3

Convert the following quantities:

a) 100 cm^3 to m^3

b) 5 dm^3 to m^3

c) 12 m^3 to cm^3

d) $0^{o}C$ to K

e) 300 K to ^{o}C

f) $34^{o}C$ to K

Exercises:

1. Calculate the volume in (m^3) occupied by one mole of a gas at 25.0 °C and 100.0 kPa.

$$V = \frac{nRT}{P} \quad V = \frac{1 \times 8.31 \times 298}{1.00 \times 10^5} = 0.0248\ m^3$$

2. Calculate the pressure of a gas (in Pa) given that 0.200 moles of the gas occupy 10.0 dm^3 at 20.0 °C.

$$P = \frac{nRT}{V} \quad P = \frac{0.200 \times 8.31 \times 293}{0.0100} = 4.87 \times 10^4\ Pa$$

3. Calculate the amount in mol of carbon dioxide which occupies 20.0 dm^3 at 27.0 °C and 100.0 kPa.

$$n = \frac{PV}{RT} \quad n = \frac{1.00 \times 10^5 \times 0.0200}{8.31 \times 300} = 0.802\ mol\ CO_2$$

4. Calculate the molar mass of a gas if a 500.0 cm^3 sample at 20.0 °C and 1.00 atm (101325 Pa) has a mass of 0.666 g.

$$M = \frac{mRT}{PV} \quad M = \frac{0.666 \times 8.31 \times 293}{101325 \times 5.00 \times 10^{-4}} = 32.0\ gmol^{-1}$$

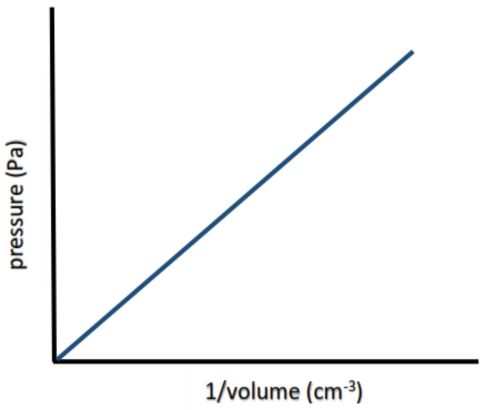
The gas laws

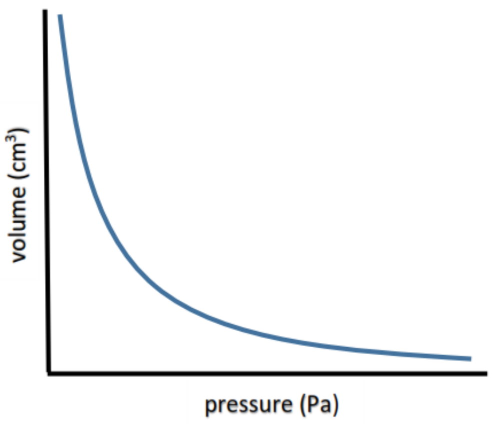
Boyle's law – the relationship between volume and pressure

- The volume occupied by a gas is inversely proportional to its pressure (at constant temperature).
- If the pressure of a fixed mass of gas is doubled (at constant temperature) then the volume of the gas will halve.

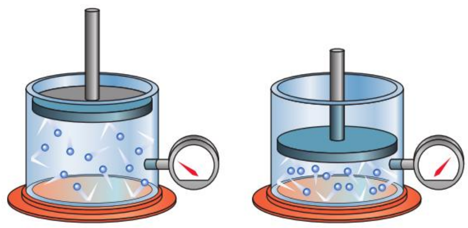
$$PV = k \quad P \propto \frac{1}{V}$$

$$P_1V_1 = P_2V_2$$





Exercise: Explain what happens to the pressure when the volume of the gas in the container is halved.

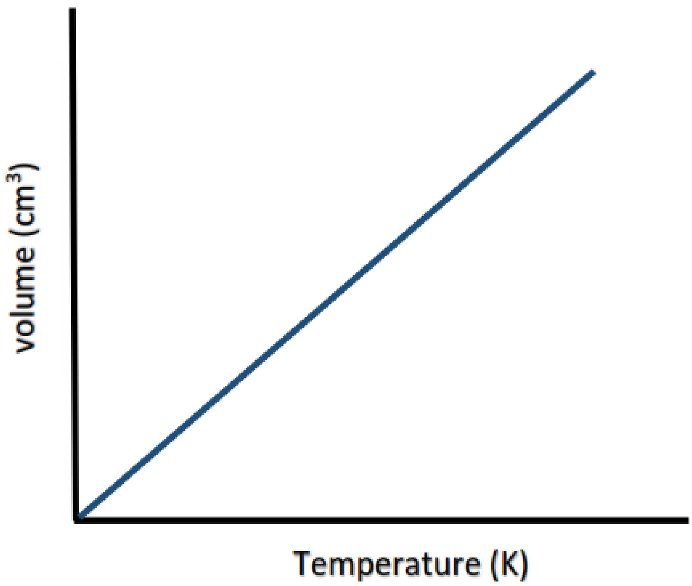


According to Boyle's law, volume and pressure are inversely proportional at constant temperature. If the volume of the gas is halved, the pressure is doubled.

Charles's law – the relationship between volume and temperature

- The volume occupied by a gas is directly proportional to its absolute temperature (at constant pressure).
- If the temperature of a fixed mass of a gas is doubled, the volume also doubles (at constant pressure).

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad V \propto T \qquad \frac{V}{T} = k$$



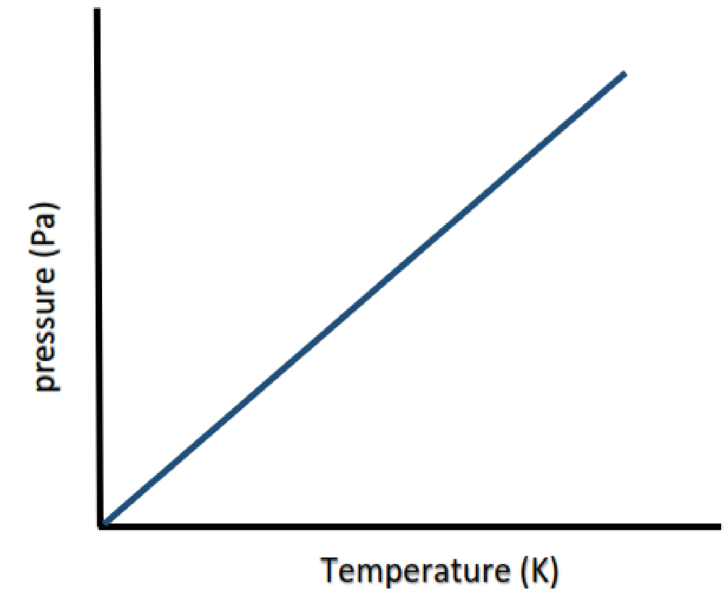
Exercise:
Imagine a balloon filled with a gas. Explain what happens when the balloon is placed into a freezer (at constant pressure).

According to Charles's law, volume and temperature are directly proportional at constant pressure. If the temperature is decreased, the volume also decreases.

Gay Lussac's law – the relationship between temperature and pressure

- The pressure of a gas is directly proportional to its absolute temperature (at constant volume).
- If the temperature of a fixed mass of gas is doubled, the pressure of the gas is also doubled.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad P \propto T \qquad \frac{P}{T} = k$$



Exercise: Explain why the pressure inside a car tyre increases on a hot day.

According to Gay Lussac's law, pressure and temperature are directly proportional at constant volume. If the temperature increases, the pressure also increases.

The combined gas law

$$P \propto T \qquad V \propto T \qquad P \propto \frac{1}{V}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Example: the molar volume of a gas is 22.7 dm^3 at STP. Calculate the volume occupied by a gas at 25^{o}C.

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$V_2 = \frac{100000 \times 22.7 \times 298}{273 \times 100000} = 24.8\ \text{dm}^3$$

- Note that this value is greater than the molar volume of a gas at 273 K (0 ^{o}C).

Additional practice examples

1) What is the final volume if the pressure of 10 dm^3 of gas is doubled at constant temperature?
   5 dm^3

2) The absolute temperature of a gas at 100.0 kPa is doubled at constant volume. What is the new pressure of the gas?
   200.0 kPa

3) The absolute temperature of 150 dm^3 of gas is doubled at constant pressure. What is the new volume of the gas?
   300 dm^3

4) What happens to the volume of a fixed mass of gas when its pressure and its absolute temperature are both doubled?
   Pressure is doubled, volume is halved;
   Temperature is doubled, volume is doubled;
   Volume is halved and then doubled, therefore the volume stays the same

5) The volume of an ideal gas at 27.0 °C is increased from 3.00 dm^3 to 6.00 dm^3. At what temperature, in °C, will the gas have the original pressure?

   Convert to kelvin (K): 27.0 + 273 = 300 K
   Volume is doubled, pressure is halved;
   Double absolute temperature (to 600 K), pressure is doubled (back to original pressure);
   Convert back to °C: 600 – 273 = 327°C